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Scheduling cyclic, non-cyclic; optimality gap, simulation

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EVALUATING THE 'OPTIMALITY GAP' BETWEEN CYCLIC AND NON-CYCLIC PLANNING POLICIES IN SUPPLY CHAINS

The evaluation of the gap between the performance measures of cyclic and non-cyclic planning policies in supply chains gives possibilities to determine efficiency a specific planning policy at the product life stages and to provide control mechanisms allowing a smooth switching from cyclic to non-cyclic planning. An overview of techniques for evaluating the 'optimality gap' in the paper focuses on techniques used in production engineering and supply chain domains, and parameters influencing the 'optimality gap'. The theoretical optimality proof on costs comparison and practical benefits based on expertise are analyzed. Benefits of the 'optimality gap' evaluation through simulation experiments are discussed. The present research presented was funded by the ECLIPS Specific Targeted Research Project of the European Commission 'Extended Collaborative integrated Life cycle supply chain Planning System' funded by EU Commission.

1. INTRODUCTION

Nowadays the selection of planning strategies, such as cyclic and non-cyclic scheduling is a topical problem for investigation within manufacturing, production engineering and supply chain management. The paper focuses on the last domain.

In case of cyclic scheduling fixed order interval lengths are applied for all items, while non-cyclic scheduling permits to have order/production intervals of varying lengths within the planning horizon. Cyclical schedules offer practical benefits in terms of easy planning and control and are preferred for the constant demand lot sizing problems. In general non-cyclic scheduling is more flexible and therefore theoretically

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more optimal than cyclic policy. When demands are dynamic, flexibility in spacing production periods permits non-cyclical schedules to result in lower total costs.

In multi-echelon environments the real life performance of a specific planning policy may differ from the theoretical one. Multi-level cyclic scheduling for long term planning that use cyclic schedules at each echelon and synchronizations with one-another, could be more efficient in practice (***www.eclipsproject.com*****).** When the gap between costs of optimal cyclic and non-cyclic schedules is small enough, theoretical optimality proof by costs comparisons may be not sufficient. Use of simulation approach allows modelling supply chain performances under different planning policies in time and by echelon, and analyzing the evolution of this gap in conditions of demand variability and uncertainty.

2. CYCLIC VERSUS NON-CYCLIC PLANNING

Prior investigations (see, [4, 5]) have shown that cyclical schedules offer practical benefits in terms of easy planning and control, but result in a higher total cost. Contrary, non-cyclic schedules are characterized as more flexible and lower in cost technique.

Evaluation of the optimality gap in supply chains mostly relates to lot sizing problems [2, 7, 8, 10], cyclic and non-cyclic planning [5, 11, 17] and cutting stock problem [18].

It is aimed to analyze the additional cost of cyclic schedule (ACCS), where the non-negative nature of this performance measure is proved. In literature [4], ACCS value or mean additional cost of cyclic solution is estimated in average by 4.4% and does not exceed 11 %. The exploration of the factors influencing the gap between total costs of cyclic and non-cyclic schedules used to estimate ACCS value, verifies [5] that the most significant factor affecting the cost gap is coefficient of demand variation (CODVAR). In literature its value is limited by 60%. In Figure 1 the cost gap is defined as a function of CODVAR, where full and dotted lines represent theoretical and real life performances of cyclic and non-cyclic planning policies, correspondingly. When 'the gap value is small enough, we can be confident that the cyclic planning policy will outperform any non-cyclic policy in practice. This is why the analysis of the “optimality gap” is crucial.

3. TECHNIQUES FOR OPTIMALITY GAP INVESTIGATION

In general, the optimality gap is defined as a percentage or ratio measure to investigate problem’s solving approaches and determine how close a solution is to optimum. Usually, the *difference of the costs* [1 - 7] expressed as percentage is used to estimate the gap value for investigated methods, i.e. cyclic and non-cyclic, optimal cyclic or non-cyclic and Lagrangian relaxed. Computational time [15], lead time for different products in terms of percentage deviation [11] and customer service levels [12] are used in order to measure the gap performance as well.

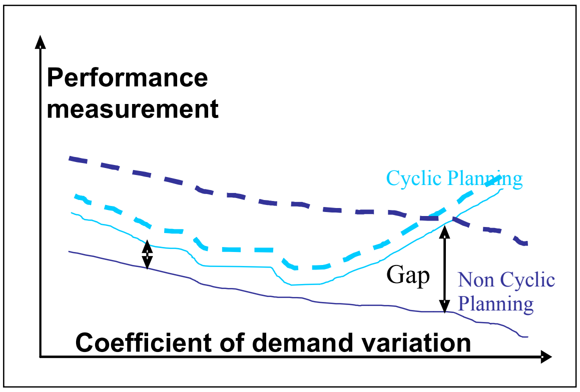

Fig. 1. Theoretic optimality versus practical benefits

To find out optimality (efficiency) of any planning policy or method, different techniques described in literature are based on:

- Theoretical Optimality Proof by Costs Comparison,
- Optimality evaluation from the Complexity Perspectives,
- Optimality evaluation by Implementation Guaranty,
- Optimality evaluation through Simulation Experiments.

Theoretical optimality proof supposes [3, 5, 9, 15, 16] existence of the *lower bound* of average cost over feasible policies used to determine the best solution. In most cases the numerical difference between solutions received by Cyclical Lagrangian Heuristic [5], Non-cyclical Lagrangian Heuristic [19], Progressive interval heuristic [7, 8], Mixed Integer programming method [2], Mixed Integer linear programming combined with continuous time [9] and a lower (optimal) bound is investigated with regard to the determination the lowest solutions gap. The main experiments are performed to compare cyclical and non-cyclical scheduling problems and estimate ACCS performance measure for multi-stage constant-demand or single-stage dynamic and deterministic demand environments. In [9] theoretical approach is also used for stochastic incapacitated lot sizing problem.

Optimality evaluation from the complexity perspectives is based on estimation of computational times [2, 15] of solving method and similarly uses the lower bound determination and gap calculation. In particularly, in [14] the worst-case computational complexity is defined on the order of (TC^M), where T is the number of time periods in the planning horizon, M is the number of items, and C represents the number of feasible combinations for an item that depends on the item cycle time and its first order period.

Optimality gap evaluation methods by Implementation Guarantee based on expertise are described in [15, 16]. As efficiency indicator percentage or ratio of the lower bound of average cost over all policies to the average cost of investigated policy is estimated. It is proved, that efficiency of q-optimal integer-ratio and optimal power-of-two policies are at least 94% and 98% that guarantee the higher efficiency of the second policy.

The optimality gap evaluation through simulation experiments in [15] is aimed to perform the sensitivity analysis within the ranges of treatment levels for significant factors. Simulation experiments revealed that coefficient of demand variation is the key factor affecting the additional cost of cyclic schedules. In [11] simulation is used to test solutions of analytical models for a stochastic economic lot scheduling problem and to compare performances of the cyclic polling policies for two-echelon multi-product supply chain with stochastic demand. The design of experiments is conducted using three parameters: demand rate, set up time and processing time.

4. PARAMETERS INFLUENCING THE OPTIMALITY GAP

The coefficient of demand variation, mentioned above, is not the only possible parameter along which it is necessary to evaluate ‘the optimality gap’. Several other parameters such as capacity utilization, setup time, time between orders, number of items and periods, are analyzed in [4, 5] and summarized in the Table 1. As gap performance measures, the additional cost of cyclic schedule (ACCS); cyclic solution cost gap (CSG) and non-cyclic solution cost gap (NSG) are introduced. As test environments more then 1000 planning problems have been solved. Experimental analysis of this problem set with 5 factors generated by full-factorial design allowed to identify significant factors (parameters) and reveal their main effects, two-factor interactions and combined effects of more then two parameters to the optimality gap.

Interaction effect of two parameters, i.e. coefficient of demand variation and capacity utilization, is illustrated in Figure 2 and 3, where ACCS Surface Plot and Contour Plot are built on the experimental results given in [5]. Based on the ACCS Contour Plot, heuristics to define sets of parameters and dependent gap values presented by ACCS contour lines could be generated and used to detect under which conditions it is suitable to switch towards or away from multi-echelon cyclic planning.

5. ‘OPTIMALITY GAP’ EVALUATION THROUGH SIMULATION

Simulation becomes the most suitable technique for analysis of the gap between performance measures of cyclic and non-cyclic planning policies in supply chains by the following reasons. Use of simulation allows to model supply chain behaviour under different planning policies in time and by echelon in conditions of demand variability and uncertainty. Corresponding periodic review or continuous review scheduling mechanisms could be implemented in the simulation model and applied for different product groups. Simulation experiments will be performed to analyse evolution of the gap in time. Several gap performance indicators and measures could be analysed at once. Simulation based analysis will allow to compare performances of different planning policies in conditions closely related to the product lifecycle dynamics, i.e. in conditions of dynamic and deterministic or stochastic demand.

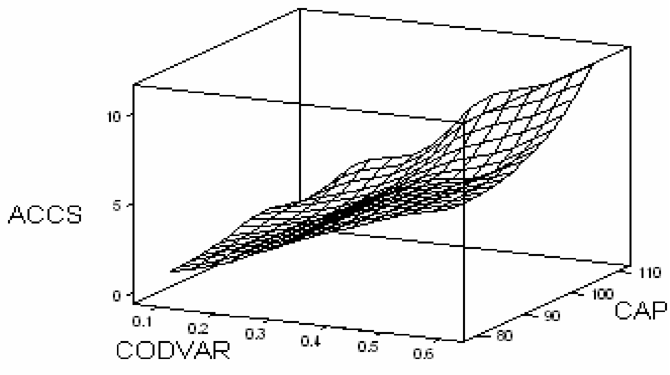

Fig. 2. ACCS Surface from CODVAR and CAP parameters

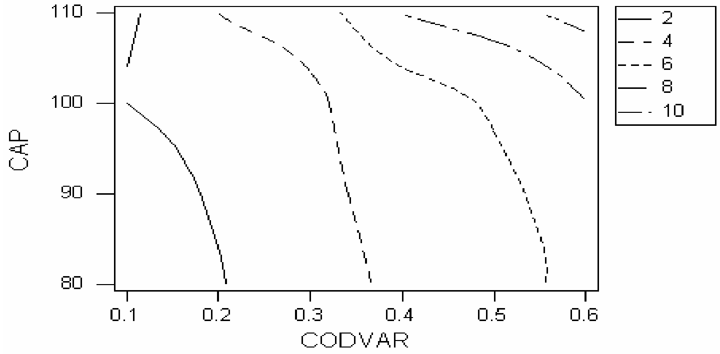

Fig. 3. ACCS Contour Plots from CODVAR and CAP parameters

Simulation-based sensitivity analysis allows learn possible effects of supply chain parameters influencing the gap performance. Interpretation and approximation of results of simulation experiments by a regression-type response surface function allows qualitatively estimate the main and interaction effects of significant factors and predict an average additional cost of cyclic schedule for different values of interest parameters. Moreover, ACCS contour plot (like in Figure 3) could be constructed directly from simulation response surface function, and as a result, heuristics that define gap dependencies on input parameters could be refined. Finally, simulation-based evaluation of the 'optimality gap' can help to define the best scheduling strategy in practice.

Table 1

Parameters' significant main, interaction and combined effects [4 -7]

Effects	Parameter(s)	Description of effects
Main effects	Coefficient of demand variation (CODVAR)	ACCS increases as CODVAR increases because of the reduction of non-cyclic solution costs. *Treatments levels from 0.1 to 0.6.*
	Capacity utilization (CAP)	Higher capacity utilization results in larger ACCS values as flexibility of non-cyclical scheduling is most valuable when capacity is more constrained (*i.e. at treatment levels 100, 110*). The effect is less strong than that of CODVAR for ACCS, and stronger for solution gaps.
	Number of items	Larger number of items decreases the variation of periodic demand and ACCS value, but the effect is less strong than that of CODVAR or CAP. *Treatments levels from 9 to 30.*
Interaction effects	CODVAR & Capacity utilization	Higher CAP makes stronger CODVAR effect to ACCS, especially to solution gaps. When high levels of both factors combined, it becomes more difficult for non-cyclic & cyclic heuristics to get solutions close to lower bounds.
	Setup time & Capacity utilization	Lower setup times have the effect similar to higher levels of CAP. The effect of lower setup times is greater at the higher CAP level, especially, with regard to solution gap. *Treatment levels randomly generated U[0.15, 0.35].*
	CODVAR & Time between orders (TBO)	The larger order intervals result in lower ACCS when there is less variation in the sum of demand over the order intervals and less variation is associated with lower CODVAR values and/or larger TBO values. As exception occurs at very low levels of TBO. *TBO treatment levels from 1 to 5 periods.*
Combined effects	Ordering cost factor, holding cost factor and CODVAR	The interaction effects of ordering cost and holding cost factors becomes more significant with increase of CODVAR that result in increase ACSS value. *Treatment levels are 0.4, 0.8; 1.6, 2.0 and 0.31, 0.75 correspondingly.*

5. CONCLUSIONS

Cyclical schedules offer practical benefits in terms of easy planning and control and are preferred in planning practice. When product demands are dynamic, flexibility in spacing production periods permits non-cyclical schedules to result in lower total costs. In multi-echelon environments the real life performance of a specific planning policy may differ from the theoretical one, and multi-level cyclic scheduling could be more efficient in practice. Applications of simulation approach allows model supply chain performances under different planning policies in time and by echelon, analyze the evolution of the gap between costs of cyclic and non-cyclic schedules in conditions of demand variability and uncertainty, and to determine efficient planning policies during the product life cycle and under changing circumstances in the supply chain system.

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